

PLASMA DYNAMICS

GENERATION OF THE LONGITUDINAL CURRENT BY THE TRANSVERSAL ELECTROMAGNETIC FIELD IN CLASSICAL AND QUANTUM PLASMAS

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From Vlasov kinetic equation for collisionless plasmas distribution function in square-law approximation on size of electromagnetic field is received. Formulas for calculation electric current at any temperature (any degree of degeneration of electronic gas) are deduced. The case of small values of the wave numbers is considered. It is shown, that the nonlinearity account leads to occurrence the longitudinal electric current directed along a wave vector. This longitudinal current orthogonal to known transversal classical current, received at the linear analysis. From the kinetic equation with Wigner integral for collisionless quantum plasma distribution function is received in square-law on vector potential approximation. Formulas for calculation electric current at any temperature are deduced. The case of small values of wave number is considered. It is shown, that size of a longitudinal current at small values of wave number and for classical plasma and for quantum plasma coincide. Graphic comparison of dimensionless size of a current quantum and classical plasma is made.

INTRODUCTION

Dielectric permeability of quantum plasma was studied by many authors [1] – [9]. It is one of the major characteristics of plasma also it is applied in the diversified questions physicists of plasma [9] – [14].

Let us notice, that in work [1] the formula for calculation of longitudinal dielectric permeability into quantum plasma for the first time has been deduced. Then the same formula has been deduced and in work [2].

In the present work formulas for calculation electric current in classical and quantum collisionless plasma at any temperature, at the any degrees of degeneration of electronic gas are deduced.

The approach developed by Klimontovich and Silin [1] is thus generalised.

At the decision of the kinetic equation we consider as in decomposition of distribution function, and in decomposition and sizes of the self-consistent electromagnetic field and Wigner integral the sizes proportional to square of intensity or potential of an external electromagnetic field.

Electric current expression consists of two composed. The first composed, linear on intensity of an electromagnetic field, is known classical expression of an electric current. This electric current is directed along the electromagnetic fields. The second composed represents an electric current, which is proportional to an intensity square of electromagnetic field. The second current it is perpendicular to the first and it is directed along the wave vector. Occurrence of the second current comes to light the spent account nonlinear character interactions of an electromagnetic field with classical and quantum plasma.

In works [15] and [16] nonlinear effects are studied into plasma. In work [16] the nonlinear current was used, in particulars, in probability questions disintegration processes. We will note, that in work [17] it is underlined existence nonlinear current along a wave vector (see the formula (2.9) from [17]).

1 CLASSICAL PLASMAS

Let us show, that in case of the classical plasma described by the Vlasov equation, the longitudinal current is generated and we will calculate its density. On existence of this current was specified more half a century ago [17]. We take the Vlasov equation describing of behaviour of collisionless plasmas

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{H}] \right) \frac{\partial f}{\partial \mathbf{p}} = 0. \quad (1.1)$$

Electric and magnetic fields are connected with the vector potential by equalities

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} = \frac{i\omega}{c} \mathbf{A}, \quad \mathbf{H} = \text{rot} \mathbf{A}.$$

Therefore,

$$\mathbf{H} = \frac{ck}{\omega} E_y \cdot (0, 0, 1), \quad [\mathbf{v}, \mathbf{H}] = \frac{ck}{\omega} E_y \cdot (v_y, -v_x, 0),$$

$$e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}, \mathbf{H}]\right)\frac{\partial f}{\partial \mathbf{p}} = \frac{e}{\omega}E_y\left[kv_y\frac{\partial f}{\partial p_x} + (\omega - kv_x)\frac{\partial f}{\partial p_y}\right].$$

Let us operate with the successive-approximations method, considering as small parametre size of intensity of electric field. Let us copy the equation (1.1) in the form

$$\begin{aligned} & \frac{\partial f^{(k)}}{\partial t} + v_x \frac{\partial f^{(k)}}{\partial x} = \\ & = -\frac{eE_y}{\omega} \left[kv_y \frac{\partial f^{(k-1)}}{\partial p_x} + (\omega - kv_x) \frac{\partial f^{(k-1)}}{\partial p_y} \right], \quad k = 1, 2. \end{aligned} \quad (1.2)$$

Here $f^{(0)} = f_0(v)$ is the absolute Fermi—Dirac distribution,

$$f_0(v) = \left[1 + \exp \frac{\mathcal{E} - \mu}{k_B T}\right]^{-1},$$

$\mathcal{E} = \frac{mv^2}{2}$ is the electrons energy, μ is the chemical potential of electronic gas, k_B is the Boltzmann constant, T is the plasma temperature.

Let us consider, that intensity of electric field varies harmoniously:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\mathbf{r} - \omega t)}.$$

Wave vector we will direct along an axis x : $\mathbf{k} = k(1, 0, 0)$, and intensity of electric field we will direct along an axis y : $\mathbf{E} = E_y(0, 1, 0)$.

We notice that

$$[\mathbf{v}, \mathbf{H}] \frac{\partial f_0}{\partial \mathbf{p}} = 0,$$

because

$$\frac{\partial f_0}{\partial \mathbf{p}} \sim \mathbf{v}.$$

We search for the solution as a first approximation in the form

$$f^{(1)} = f_0(P) + f_1,$$

where $f_1 \sim E_y$.

In this approximation the equation (1.2) becomes simpler

$$\frac{\partial f_1}{\partial t} + v_x \frac{\partial f_1}{\partial x} = -eE_y \frac{\partial f_0}{\partial p_y}. \quad (1.3)$$

From (1.3) we receive

$$f_1 = \frac{2ieE_y}{p_T} \frac{P_y g(P)}{\omega - kv_T P_x}, \quad (1.4)$$

where

$$g(P) = \frac{e^{P^2 - \alpha}}{(1 + e^{P^2 - \alpha})^2}, \quad \mathbf{P} = \frac{\mathbf{p}}{p_T}.$$

Here $v_T = \sqrt{2k_B T/m}$ is the thermal electrons velocity, $p_T = mv_T$ is the thermal electrons momentum, \mathbf{P} is the dimensionless momentum.

In the second approximation for the solution of the equation (1.2) we search in the form

$$f^{(2)} = f^{(1)} + f_2,$$

where $f_2 \sim E_y^2$.

From the equation (1.2) it is found

$$f_2 = \frac{e^2 E_y^2}{p_T^2 \omega} \left[kv_T P_y^2 \frac{\partial}{\partial P_x} \left(\frac{g(P)}{\omega - kv_T P_x} \right) + \frac{\partial(P_y g(P))}{\partial P_y} \right] \frac{1}{\omega - kv_T P_x}. \quad (1.5)$$

Distribution function in the second approximation across the field is constructed

$$f = f^{(2)} = f^{(0)} + f_1 + f_2, \quad (1.6)$$

where f_1, f_2 are given by equalities (1.4) and (1.6).

Let us find electric current density

$$\mathbf{j} = e \int \mathbf{v} f \frac{2d^3 p}{(2\pi\hbar)^3}. \quad (1.7)$$

From equalities (1.4) – (1.6) it is visible, that the vector of density of a current has two nonzero components

$$\mathbf{j} = (j_x, j_y, 0).$$

Here j_y is the density of transversal current,

$$j_y = e \int v_y f \frac{2d^3 p}{(2\pi\hbar)^3} = e \int v_y f_1 \frac{2d^3 p}{(2\pi\hbar)^3}. \quad (1.8)$$

This current is directed along an electromagnetic field, its density it is defined only by the first approximation of function of distribution. The second approximation of function of distribution the contribution to current density does not bring.

The density of the transversal current is defined by equality

$$j_y = \frac{ie^2 k_T^3}{2\pi^3 m} E_y(x, t) \int \frac{P_y^2 g(P) d^3 P}{\omega - kv_T P_x}.$$

For density of the longitudinal current according to its definition it is had

$$j_x = e \int v_x f \frac{2d^3 p}{(2\pi\hbar)^3} = e \int v_x f_2 \frac{2d^3 p}{(2\pi\hbar)^3} = \frac{2ev_T p_T^3}{(2\pi\hbar)^3} \int P_x f_2 d^3 P.$$

By means of (1.6) from here it is received, that

$$j_x = e^3 E_y^2 \frac{2mv_T^2}{(2\pi\hbar)^3 \omega} \int \left[\frac{\partial(P_y g(P))}{\partial P_y} + kv_T P_y^2 \frac{\partial}{\partial P_x} \left(\frac{g(P)}{\omega - kv_T P_x} \right) \right] \frac{P_x d^3 P}{\omega - kv_T P_x}. \quad (1.9)$$

In the first integral from (1.9) internal integral on P_y it is equal to zero. In the second integral from (1.9) internal integral on P_x it is calculated in parts

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial P_x} \left(\frac{g(P)}{\omega - kv_T P_x} \right) \frac{P_x dP_x}{\omega - kv_T P_x} = -\omega \int_{-\infty}^{\infty} \frac{g(P) dP_x}{(\omega - kv_T P_x)^3}.$$

Hence, equality (1.9) becomes simpler

$$j_x = -e^3 E_y^2 \frac{2mv_T^3 k}{(2\pi\hbar)^3} \int \frac{g(P) P_y^2 d^3 P}{(\omega - kv_T P_x)^3} = \frac{e^3 E_y^2 q}{4\pi^3 \hbar m v_T^2} \int \frac{g(P) P_y^2 d^3 P}{(qP_x - \Omega)^3}. \quad (1.10)$$

Here

$$\Omega = \frac{\omega}{k_T v_T}, \quad q = \frac{k}{k_T}.$$

Equality (1.10) is reduced to one-dimensional integral

$$j_x = \frac{e^3 E_y^2 q}{8\pi^2 \hbar m v_T^2} \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - P_x^2}) dP_x}{(qP_x - \Omega)^3}. \quad (1.11)$$

Let us find numerical density (concentration) of particles of the plasma, corresponding to Fermi–Dirac distribution

$$N = \int f_0(P) \frac{2d^3p}{(2\pi\hbar)^3} = \frac{8\pi p_T^3}{(2\pi\hbar)^3} \int_0^\infty \frac{e^{\alpha-P^2} P^2 dP}{1 + e^{\alpha-P^2}} = \frac{k_T^3}{2\pi^2} l_0(\alpha),$$

where k_T is the thermal wave number, $k_T = \frac{mv_T}{\hbar}$,

$$l_0(\alpha) = \int_0^\infty \ln(1 + e^{\alpha-P^2}) dP.$$

In expression before integral from (1.11) we will allocate the plasma (Langmuir) frequency

$$\omega_p = \sqrt{\frac{4\pi e^2 N}{m}}$$

and numerical density (concentration) N , and last we will express through thermal wave number. We will receive

$$j_x^{\text{long}} = E_y^2 \frac{e\Omega_p^2}{p_T} \frac{q}{16\pi l_0(\alpha)} \int_{-\infty}^\infty \frac{\ln(1 + e^{\alpha-\tau^2}) d\tau}{(q\tau - \Omega)^3},$$

where $\Omega_p = \frac{\omega_p}{k_T v_T} = \frac{\hbar\omega_p}{mv_T^2}$ is the dimensionless plasma frequency.

This equality we will copy in the form

$$j_x^{\text{long}} = J_{\text{classic}}(\Omega, q) \sigma_{l,tr} k E_y^2, \quad (1.12)$$

where $\sigma_{l,tr}$ is the longitudinal–transversal conductivity, $J_{\text{classic}}(\Omega, q)$ is the dimensionless part of current,

$$\sigma_{l,tr} = \frac{e\hbar}{p_T^2} \left(\frac{\hbar\omega_p}{mv_T^2} \right)^2 = \frac{e}{p_T k_T} \Omega_p^2,$$

$$J_{\text{classic}}(\Omega, q) = \frac{1}{16\pi l_0(\alpha)} \int_{-\infty}^\infty \frac{\ln(1 + e^{\alpha-\tau^2}) d\tau}{(q\tau - \Omega)^3}.$$

The integral from dimensionless part of current is calculated according to known Landau rule

$$\int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q\tau - \Omega)^3} = -i \frac{\pi}{2q^3} \left[\ln(1 + e^{\alpha - \tau^2}) \right]'' \Big|_{\tau=\Omega/q} + \\ + \text{V.p.} \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q\tau - \Omega)^3}.$$

Symbol V.p. before integral means, that integral it is understood in sense of a principal value.

Let us introduce the transversal electromagnetic field

$$\mathbf{E}_{\text{tr}} = \mathbf{E} - \frac{\mathbf{k}(\mathbf{E}\mathbf{k})}{k^2} = \mathbf{E} - \frac{\mathbf{q}(\mathbf{E}\mathbf{q})}{q^2}.$$

Equality (1.12) can be written down in the invariant form

$$\mathbf{j}^{\text{long}} = J_{\text{classic}}(\Omega, q) \sigma_{l, tr} \mathbf{k} \mathbf{E}_{tr}^2.$$

Let us pass to consideration of the case of small values of wave number. From expression (1.10) at small values of wave number it is received

$$j_x^{\text{classic}} = -\frac{2e^3 E_y^2 m v_T^3 k}{(2\pi\hbar)^3 \omega^3} \int g(P) P_y^2 d^3 P = -\frac{e^3 E_y^2 k_T^3 l_0(\alpha)}{4\pi^2 \omega^3} k = \\ = -\frac{1}{8\pi} \cdot \frac{e}{m\omega} \left(\frac{\omega_p}{\omega} \right)^2 k E_y^2. \quad (1.13)$$

2 KINETIC EQUATION FOR WIGNER FUCTION

Shrödinger equation

$$i\hbar \frac{\partial \rho}{\partial t} = H\rho - H^{*'}\rho$$

for density matrix ρ under condition of calibration $\text{div} \mathbf{A} = 0$ it will be transformed in the kinetic equation [18]

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + W[f] = 0, \quad (2.1)$$

written down concerning quantum distribution Wigner function

$$f(\mathbf{r}, \mathbf{p}, t) = \int \rho(\mathbf{r} + \frac{\mathbf{a}}{2}, \mathbf{r} - \frac{\mathbf{a}}{2}, t) e^{-i\mathbf{p}\mathbf{a}/\hbar} d^3a,$$

besides

$$\rho(\mathbf{R}, \mathbf{R}', t) = \frac{1}{(2\pi\hbar)^3} \int f(\frac{\mathbf{R} + \mathbf{R}'}{2}, \mathbf{p}, t) e^{i\mathbf{p}(\mathbf{R} - \mathbf{R}')/\hbar} d^3p.$$

Here H is the Hamilton operator, H^* is the complex conjugated to H operator, $H^{*'}$ is the complex conjugated to H operator, acting on the shaded spatial variables \mathbf{r}' . The scalar potential is considered equal to zero. Integral of Wigner is equal (see, example, [11]):

$$\begin{aligned} W[f] = & \iint \left\{ -\frac{e}{2mc} \left[\mathbf{A}(\mathbf{r} + \frac{\mathbf{a}}{2}, t) + \mathbf{A}(\mathbf{r} - \frac{\mathbf{a}}{2}, t) - 2\mathbf{A}(\mathbf{r}, t) \right] \frac{\partial f}{\partial \mathbf{r}} - \right. \\ & \left. -\frac{ie}{mc\hbar} \left[\mathbf{A}(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - \mathbf{A}(\mathbf{r} - \frac{\mathbf{a}}{2}, t) \right] \mathbf{p}' f + \right. \\ & \left. + \frac{ie^2}{2mc^2\hbar} \left[\mathbf{A}^2(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - \mathbf{A}^2(\mathbf{r} - \frac{\mathbf{a}}{2}, t) \right] f \right\} e^{i(\mathbf{p}' - \mathbf{p})\mathbf{a}/\hbar} \frac{d^3a d^3p'}{(2\pi\hbar)^3}. \end{aligned}$$

Vector potential of an electromagnetic field we take orthogonal to direction of the wave vector \mathbf{k} ($\mathbf{k}\mathbf{A} = 0$) in the form of running harmonious wave

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_0 e^{i(\mathbf{k}\mathbf{r} - \omega t)}.$$

We transform the previous expression of Wigner integral (see, example, [11]). We find that

$$\begin{aligned} W[f] = & -\mathbf{A}(\mathbf{r}, t) \frac{e}{2mc} \left[\nabla f(\mathbf{r}, \mathbf{p} - \frac{\hbar\mathbf{k}}{2}, t) + \nabla f(\mathbf{r}, \mathbf{p} + \frac{\hbar\mathbf{k}}{2}, t) - 2\nabla f(\mathbf{r}, \mathbf{p}, t) \right] - \\ & -\mathbf{A}(\mathbf{r}, t) \frac{ie}{mc\hbar} \left\{ \mathbf{p} \left[f(\mathbf{r}, \mathbf{p} - \frac{\hbar\mathbf{k}}{2}, t) - f(\mathbf{r}, \mathbf{p} + \frac{\hbar\mathbf{k}}{2}, t) \right] + \right. \\ & \left. + \mathbf{A}^2(\mathbf{r}, t) \frac{ie^2}{2mc^2\hbar} \left[f(\mathbf{r}, \mathbf{p} - \hbar\mathbf{k}, t) - f(\mathbf{r}, \mathbf{p} + \hbar\mathbf{k}, t) \right] \right\}. \end{aligned} \quad (2.2)$$

Let us enter local and absolute distributions of Fermi—Dirac

$$f^{(0)} = f_0(\mathbf{r}, \mathbf{C}, t) = [1 + \exp(C^2 - \alpha)]^{-1},$$

and

$$f^{(0)} = f_0(P) = [1 + \exp(P^2 - \alpha)]^{-1}.$$

Here

$$\mathbf{C} \equiv \mathbf{C}(\mathbf{r}, \mathbf{P}, t) = \frac{\mathbf{v}}{v_T} = \mathbf{P} - \frac{e}{cp_T} \mathbf{A}(\mathbf{r}, t), \quad \alpha = \frac{\mu}{k_B T},$$

\mathbf{C} is the dimensionless electrons velocity, $v_T = 1/\sqrt{\beta}$ is the thermal electrons velocity, $\beta = m/2k_B T$, $\mathbf{P} = \mathbf{p}/p_T$ is the dimensionless electrons momentum, m is the electron mass, k_B is the Boltzmann constant, T is the plasma temperature, μ is the chemical potential of electronical gas, α is the dimensionless chemical potential.

Let us show, that the first composed in Wigner integral (2.2) equals to zero. We will notice, that according to problem statement gradient of quantum distribution function it is proportional to the vector \mathbf{k} : $\nabla f \sim \mathbf{k}$. Therefore

$$\mathbf{A}(\mathbf{r}, t) \left[\nabla f(\mathbf{r}, \mathbf{p} - \frac{\hbar \mathbf{k}}{2}, t) + \nabla f(\mathbf{r}, \mathbf{p} + \frac{\hbar \mathbf{k}}{2}, t) - 2\nabla f(\mathbf{r}, \mathbf{p}, t) \right] \sim \mathbf{A} \mathbf{k} = 0.$$

Thus, Wigner integral is equal

$$\begin{aligned} W[f] = & \mathbf{p} \mathbf{A} \frac{ie}{mc\hbar} \left[f(\mathbf{r}, \mathbf{p} + \frac{\hbar \mathbf{k}}{2}, t) - f(\mathbf{r}, \mathbf{p} - \frac{\hbar \mathbf{k}}{2}, t) \right] + \\ & - \mathbf{A}^2 \frac{ie^2}{2mc^2\hbar} \left[f(\mathbf{r}, \mathbf{p} + \hbar \mathbf{k}, t) - f(\mathbf{r}, \mathbf{p} - \hbar \mathbf{k}, t) \right]. \end{aligned}$$

Let us return to the kinetic equation (2.1). We will consider convectional derivative from this equation

$$\mathbf{v} \frac{\partial f}{\partial \mathbf{r}} = \left(\frac{\mathbf{p}}{m} - \frac{e}{mc} \mathbf{A}(\mathbf{r}, t) \right) \frac{\partial f}{\partial \mathbf{r}}.$$

Thanks to conditions

$$\frac{\partial f}{\partial \mathbf{r}} \sim \mathbf{k}, \quad \mathbf{A} \mathbf{k} = 0$$

we receive that

$$\mathbf{v} \frac{\partial f}{\partial \mathbf{r}} = \frac{\mathbf{p}}{m} \frac{\partial f}{\partial \mathbf{r}} = v_T \mathbf{P} \frac{\partial f}{\partial \mathbf{r}}.$$

Let us solve further kinetic Wigner equation for quantum distributions function

$$\begin{aligned} \frac{\partial f}{\partial t} + v_T \mathbf{P} \frac{\partial f}{\partial \mathbf{r}} + \frac{iev_T \mathbf{P} \mathbf{A}}{c\hbar} \left[f(\mathbf{r}, \mathbf{P} + \frac{\mathbf{q}}{2}, t) - f(\mathbf{r}, \mathbf{P} - \frac{\mathbf{q}}{2}, t) \right] - \\ - \mathbf{A}^2 \frac{ie^2}{2mc^2\hbar} \left[f(\mathbf{r}, \mathbf{P} + \mathbf{q}, t) - f(\mathbf{r}, \mathbf{P} - \hbar\mathbf{q}, t) \right] = 0. \end{aligned} \quad (2.3)$$

Here

$$\mathbf{q} = \frac{\hbar \mathbf{k}}{p_T} = \frac{\mathbf{k}}{k_T}, \quad k_T = \frac{p_T}{\hbar},$$

k_T is the thermal wave number, \mathbf{q} is the dimensionless wave number.

3 SOLUTION OF WIGNER EQUATION

Let us consider as small parametre size of vector potential of electromagnetic field $\mathbf{A}(\mathbf{r}, t)$. The solution of the equations (2.3) we will be to search by the method of consecutive approximations.

As the first approach for the solution we search in the form, linear on vector potential concerning to absolute distribution of Fermi–Dirac:

$$f^{(1)} = f_0(P) + f_1, \quad f_1 \sim \mathbf{A}(\mathbf{r}, t). \quad (3.1)$$

As the first approximation we take linear concerning to vector potential a part of Wigner equation

$$\frac{\partial f}{\partial t} + v_T \mathbf{P} \frac{\partial f}{\partial \mathbf{r}} + \frac{iev_T \mathbf{P} \mathbf{A}}{m\hbar} \left[f_0(\mathbf{P} + \frac{\mathbf{q}}{2}) - f_0(\mathbf{P} - \frac{\mathbf{q}}{2}) \right] = 0. \quad (3.2)$$

Here

$$f_0(\mathbf{P} \pm \frac{\mathbf{q}}{2}) = \left[1 + \exp \left[\left(\mathbf{P} \pm \frac{\mathbf{q}}{2} \right)^2 - \alpha \right] \right]^{-1}.$$

Substituting (3.1) in (3.2), we receive the equation

$$-i(\omega - v_T \mathbf{k} \mathbf{P}) f_1 = -\frac{iev_T}{c\hbar} \mathbf{P} \mathbf{A} \left[f_0(\mathbf{P} + \frac{\mathbf{q}}{2}) - f_0(\mathbf{P} - \frac{\mathbf{q}}{2}) \right].$$

From this equation we receive

$$f_1 = \frac{ev_T}{c\hbar} \mathbf{P} \mathbf{A} \frac{f_0(\mathbf{P} + \mathbf{q}/2) - f_0(\mathbf{P} - \mathbf{q}/2)}{\omega - v_T \mathbf{k} \mathbf{P}}. \quad (3.3)$$

Hence, as a first approximation according to (3.1) and (3.3) the solution it is constructed

$$f^{(1)} = f_0(P) + \frac{ev_T}{c\hbar} \mathbf{P} \mathbf{A} \frac{f_0(\mathbf{P} + \mathbf{q}/2) - f_0(\mathbf{P} - \mathbf{q}/2)}{\omega - v_T \mathbf{k} \mathbf{P}}. \quad (3.4)$$

In the second approach we search for the solution in the form

$$f^{(2)} = f^{(1)} + f_2, \quad f_2 \sim \mathbf{A}^2(\mathbf{r}, t).$$

In the equation (2.3) in the first square bracket function f we will replace on $f^{(1)}$, and in the second square bracket function f we will replace on f_0 , i.e. $f^{(2)}$ we search from the equation

$$\begin{aligned} \frac{\partial f^{(2)}}{\partial t} + v_T \mathbf{P} \frac{\partial f^{(2)}}{\partial \mathbf{r}} + \frac{iev_T \mathbf{P} \mathbf{A}}{c\hbar} \left[f^{(1)}(\mathbf{r}, \mathbf{P} + \frac{\mathbf{q}}{2}, t) - f^{(1)}(\mathbf{r}, \mathbf{P} - \frac{\mathbf{q}}{2}, t) \right] - \\ - \mathbf{A}^2 \frac{ie^2}{2mc^2\hbar} \left[f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q}) \right] = 0. \end{aligned} \quad (3.5)$$

In this equation according to (3.4)

$$\begin{aligned} f^{(1)}\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) &= f_0\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) + \frac{ev_T}{c\hbar} \left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) \mathbf{A} \frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)}, \\ f^{(1)}\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) &= f_0\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) + \frac{ev_T}{c\hbar} \left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \mathbf{A} \frac{f_0(P) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)}, \end{aligned}$$

We notice that

$$\mathbf{A} \mathbf{q} = 0, \quad \text{because} \quad \mathbf{A} \mathbf{q} \sim \mathbf{A} \mathbf{k} = 0.$$

Therefore the previous two parities become simpler

$$\begin{aligned} f^{(1)}\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) &= f_0\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) + \frac{ev_T}{c\hbar} (\mathbf{P} \mathbf{A}) \frac{f_0(P) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)}, \\ f^{(1)}\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) &= f_0\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) + \frac{ev_T}{c\hbar} (\mathbf{P} \mathbf{A}) \frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)}. \end{aligned}$$

The equation (3.5) we will copy in an explicit form

$$\begin{aligned} & \frac{\partial f^{(1)}}{\partial t} + v_T \mathbf{P} \frac{\partial f^{(1)}}{\partial \mathbf{r}} + \frac{iev_T \mathbf{P} \mathbf{A}}{c\hbar} \left[f_0(\mathbf{P} + \frac{\mathbf{q}}{2}) - f_0(\mathbf{P} - \frac{\mathbf{q}}{2}) \right] + \\ & + \frac{\partial f_2}{\partial t} + v_T \mathbf{P} \frac{\partial f_2}{\partial \mathbf{r}} + i \frac{e^2 v_T^2 (\mathbf{P} \mathbf{A})^2}{c^2 \hbar^2} \left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)} - \right. \\ & \left. - \frac{f_0(P) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)} \right] - \frac{ie^2 \mathbf{A}^2}{2mc^2 \hbar} \left[f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q}) \right] = 0. \end{aligned}$$

First three composed in this equation give zero agree to the equation (3.2). The rest part of equation leads to equality

$$\begin{aligned} -2i(\omega - v_T \mathbf{k} \mathbf{P}) f_2 = & -i \frac{e^2 v_T^2 (\mathbf{P} \mathbf{A})^2}{c^2 \hbar^2} \left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)} - \right. \\ & \left. - \frac{f_0(P) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)} \right] + \frac{ie^2 \mathbf{A}^2}{2mc^2 \hbar} \left[f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q}) \right]. \end{aligned}$$

From here we find that

$$\begin{aligned} f_2 = & \frac{e^2 v_T^2 (\mathbf{P} \mathbf{A})^2}{2c^2 \hbar^2 (\omega - v_T \mathbf{k} \mathbf{P})} \left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)} - \right. \\ & \left. - \frac{f_0(P) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)} \right] - \frac{e^2 \mathbf{A}^2}{4mc^2 \hbar} \frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k} \mathbf{P}}. \end{aligned} \quad (3.6)$$

So, quantum distribution function of Wigner is constructed and it is defined by equalities (3.1), (3.3) and (3.6): $f = f_0(P) + f_1 + f_2$, or, is more detailed

$$\begin{aligned} f = & f_0(P) + \frac{ev_T}{c\hbar} \mathbf{P} \mathbf{A} \frac{f_0(\mathbf{P} + \mathbf{q}/2) - f_0(\mathbf{P} - \mathbf{q}/2)}{\omega - v_T \mathbf{k} \mathbf{P}} + \frac{e^2 v_T^2 (\mathbf{P} \mathbf{A})^2}{2c^2 \hbar^2 (\omega - v_T \mathbf{k} \mathbf{P})} \times \\ & \times \left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)} + \frac{f_0(\mathbf{P} - \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)} \right] - \\ & - \frac{e^2 \mathbf{A}^2}{4mc^2 \hbar} \frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k} \mathbf{P}}. \end{aligned}$$

4 ELECTRICAL CURRENT IN QUANTUM PLASMA

By definition, the electric current density is equal

$$\mathbf{j}(\mathbf{r}, t) = e \int \mathbf{v}(\mathbf{r}, \mathbf{p}, t) f(\mathbf{r}, \mathbf{p}, t) \frac{2 d^3 p}{(2\pi\hbar)^3}. \quad (4.1)$$

Substituting in equality (4.1) obvious expression for velocity

$$\mathbf{v}(\mathbf{r}, \mathbf{P}, t) = v_T \mathbf{P} - \frac{e \mathbf{A}(\mathbf{r}, t)}{mc},$$

and distribution function according to equality $f = f_0(P) + f_1 + f_2$.

Leaving linear and square-law expressions concerning vector potential of a field, we receive

$$\begin{aligned} \mathbf{j} = & \frac{2ep_T^3}{(2\pi\hbar)^3} \int \left[v_T \mathbf{P} f_1 - \frac{e}{mc} \mathbf{A} f_0(P) \right] d^3 P + \\ & + \frac{2ep_T^3}{(2\pi\hbar)^3} \int \left[v_T \mathbf{P} f_2 - \frac{e}{mc} \mathbf{A} f_1 \right] d^3 P. \end{aligned} \quad (4.2)$$

Let us show, that the formula (4.2) for electric current density contains two nonzero components: $\mathbf{j} = (j_x, j_y, 0)$. One component j_y it is linear on potential of an electromagnetic field and is directed lengthways field. It is the known formula for electric current density, so-called "transversal current". The second component j_x is quadratic on potential of field also and it is directed along the wave vector. It is "longitudinal current".

The first composed in (4.2) is linear on vector potential expression, and second is square-law. We will write out these composed in obvious form

$$\mathbf{j}^{\text{linear}} = \frac{2ep_T^3}{(2\pi\hbar)^3} \int \left[\frac{ev_T^2}{c\hbar} \mathbf{P}(\mathbf{P}\mathbf{A}) \frac{f_0(\mathbf{P} + \mathbf{q}/2) - f_0(\mathbf{P} - \mathbf{q}/2)}{\omega - v_T \mathbf{k}\mathbf{P}} - \frac{e\mathbf{A}}{mc} f_0(P) \right] d^3 P \quad (4.3)$$

and

$$\mathbf{j}^{\text{quadr}} = \frac{2ep_T^3}{(2\pi\hbar)^3} \int \left[- \frac{e^2 v_T \mathbf{A}(\mathbf{P}\mathbf{A})}{mc^2 \hbar} \left[f_0(\mathbf{P} + \frac{\mathbf{q}}{2}) - f_0(\mathbf{P} - \frac{\mathbf{q}}{2}) \right] + \right.$$

$$\begin{aligned}
& + \frac{e^2 v_T^3 \mathbf{P}(\mathbf{P}\mathbf{A})^2}{2c^2 \hbar^2} \left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)} - \frac{f_0(P) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)} \right] - \\
& - \frac{e^2 v_T \mathbf{P}\mathbf{A}^2}{4mc^2 \hbar} [f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q})] \left] \frac{d^3 P}{\omega - v_T \mathbf{k}\mathbf{P}}. \quad (4.4)
\end{aligned}$$

Expression (4.3) is linear expression of the electric current, found, in particular, in our previous work [11]. This vector expression contains only one component, directed along the electromagnetic fields. Really, if a wave vector to direct along an axis x i.e. to take $\mathbf{k} = k(1, 0, 0)$, and potential electromagnetic fields to direct along an axis y , i.e. to take

$$\mathbf{A}(\mathbf{r}, t) = (0, A_y(x, t), 0),$$

from the formula (4.3) we receive

$$j_y^{\text{linear}} = -\frac{2e^2 p_T^3 A_y}{(2\pi \hbar)^3 mcq} \int \left(\frac{f_0(P_x + q/2) - f_0(P_x - q/2)}{P_x - \Omega/q} P_y^2 + q f_0(P) \right) d^3 P. \quad (4.5)$$

Here

$$f_0(P_x \pm q/2) = \left[1 + e^{(P_x \pm q/2)^2 + P_y^2 + P_z^2 - \alpha} \right]^{-1}, \quad \Omega = \frac{\omega}{k_T v_T}.$$

Let us consider expression for an electric current (4.4), proportional to a square of potential of an electromagnetic field. Let us notice, that the first composed in this expression is equal to zero. Hence, this expression becomes simpler

$$\begin{aligned}
\mathbf{j}^{\text{quadr}} = & \frac{2ep_T^3}{(2\pi \hbar)^3} \int \left[\frac{e^2 v_T^3 \mathbf{P}(\mathbf{P}\mathbf{A})^2}{2c^2 \hbar^2 (\omega - v_T \mathbf{k}\mathbf{P})} \left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)} + \right. \right. \\
& \left. \left. + \frac{f_0(\mathbf{P} - \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)} \right] - \frac{e^2 v_T \mathbf{P}\mathbf{A}^2}{4mc^2 \hbar} \frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}\mathbf{P}} \right] d^3 P. \quad (4.6)
\end{aligned}$$

Let us notice, that vector expression (4.6) contains one nonzero of the electric current component, directed along the wave vector

$$j_x^{\text{quadr}} = \frac{e^3 p_T^3 A_y^2}{(2\pi \hbar)^3 c^2 m^2 v_T} \int \left[\left[\frac{f_0(P_x + q) - f_0(P)}{qP_x + q^2/2 - \Omega} + \frac{f_0(P_x - q) - f_0(P)}{qP_x - q^2/2 - \Omega} \right] P_y^2 + \right.$$

$$+\frac{1}{2}[f_0(P_x + q) - f_0(P_x - q)]\left]\frac{P_x d^3 P}{qP_x - \Omega}.\quad (4.7)$$

Let us lead to a kind convenient for calculations, the formula (4.7) for density of a longitudinal current.

Let's consider the first integral from (4.7). We will calculate the internal integrals in plane (P_y, P_z) , passing to polar coordinates

$$\int f_0(P_x \pm q) P_y^2 dP_y dP_z = \pi \int_0^\infty \rho \ln(1 + e^{\alpha - (P_x \pm q)^2 - \rho^2}) d\rho,$$

$$\int f_0(P_x \pm q) dP_y dP_z = \pi \ln(1 + e^{\alpha - (P_x \pm q)^2}).$$

Thus, size of density of generated longitudinal current into quantum plasma it is equal

$$j_x^{\text{quant}} = \frac{\pi e^3 p_T^3 A_y^2}{(2\pi\hbar)^3 m^2 c^2 v_T} \int_{-\infty}^\infty \left[\frac{L(P_x + q, P_x)}{qP_x + q^2/2 - \Omega} + \frac{L(P_x - q, P_x)}{qP_x - q^2/2 - \Omega} + \right. \\ \left. + \frac{1}{2} \ln \frac{1 + e^{\alpha - (P_x + q)^2}}{1 + e^{\alpha - (P_x - q)^2}} \right] \frac{P_x dP_x}{qP_x - \Omega}. \quad (4.8)$$

Here

$$L(P_x \pm q, P_x) = \int_0^\infty \rho \ln \frac{1 + e^{\alpha - (P_x \pm q)^2 - \rho^2}}{1 + e^{\alpha - P_x^2 - \rho^2}} d\rho.$$

Let us transform the expression standing in integral (4.8). At first let us pass from potential to intensity of a field $A_y = -(ic/\omega)E_y$. We will receive

$$\frac{\pi e^3 p_T^3 A_y^2}{(2\pi\hbar)^3 m^2 c^2 v_T} = -\frac{\pi e^3 k_T^3 E_y^2}{8\pi^3 m^2 v_T \omega^2}.$$

Let us transform this expression by means of expression for the thermal wave numbers k_T . We receive, that

$$-\frac{\pi e^3 k_T^3 E_y^2}{8\pi^3 m^2 v_T \omega^2} = -\frac{e^3 N E_y^2}{4l_0(\alpha) m^2 v_T \omega^2} = -\frac{1}{16\pi} \cdot \frac{e}{m v_T} \left(\frac{\omega_p}{\omega}\right)^2 E_y^2 =$$

$$= -\frac{1}{16\pi} \frac{e}{p_T} \left(\frac{\Omega_p}{\Omega} \right)^2 E_y^2 = -\frac{1}{16\pi l_0(\alpha) \Omega^2} \frac{e \Omega_p^2}{p_T} E_y^2 = -\frac{1}{16\pi l_0(\alpha) q \Omega^2} \sigma_{l,tr} k E_y^2,$$

where the quantity of longitudinal-transversal conductivity $\sigma_{l,tr}$ was introduced in part 1: $\sigma_{l,tr} = e \Omega_p^2 / (p_T k_T)$.

Now equality (4.8) we will present in the form

$$j_x^{\text{quant}} = J_{\text{quant}}(\Omega, q) \sigma_{l,tr} k E_y^2(x, t). \quad (4.9)$$

In (4.9) $J_{\text{quant}}(\Omega, q)$ is the density of dimensionless longitudinal current,

$$J_{\text{quant}}(\Omega, q) = -\frac{1}{16\pi l_0(\alpha) q \Omega^2} \int_{-\infty}^{\infty} \left[\frac{L(\tau + q, \tau)}{q\tau + q^2/2 - \Omega} + \frac{L(\tau - q, \tau)}{q\tau - q^2/2 - \Omega} + \right. \\ \left. + \frac{1}{2} \ln \frac{1 + e^{\alpha - (\tau + q)^2}}{1 + e^{\alpha - (\tau - q)^2}} \right] \frac{\tau d\tau}{q\tau - \Omega}. \quad (4.10)$$

Let us transform integral from (4.10) from the first composed

$$J_1 = \int_{-\infty}^{\infty} \left[\frac{L(\tau + q, \tau)}{q\tau + q^2/2 - \Omega} + \frac{L(\tau - q, \tau)}{q\tau - q^2/2 - \Omega} \right] \frac{\tau d\tau}{q\tau - \Omega} = J_2 + J_3.$$

Here

$$J_2 = \int_{-\infty}^{\infty} \frac{L(\tau + q, \tau) \tau d\tau}{(q\tau + q^2/2 - \Omega)(q\tau - \Omega)}, \\ J_3 = \int_{-\infty}^{\infty} \frac{L(\tau - q, \tau) \tau d\tau}{(q\tau - q^2/2 - \Omega)(q\tau - \Omega)}.$$

In integral J_2 we will make variable replacement $\tau \rightarrow \tau - q/2$, and in integral J_3 we will make replacement $\tau \rightarrow \tau + q/2$. As result it is received

$$J_2 = \int_{-\infty}^{\infty} \frac{L(\tau + q/2, \tau - q/2) (\tau - q/2) d\tau}{(q\tau - \Omega)(q\tau - q^2/2 - \Omega)}, \\ J_3 = \int_{-\infty}^{\infty} \frac{L(\tau - q/2, \tau + q/2) (\tau + q/2) d\tau}{(q\tau - \Omega)(q\tau + q^2/2 - \Omega)}.$$

We notice that

$$\begin{aligned}
L(\tau - q/2, \tau + q/2) &= \int_0^\infty \rho \ln \frac{1 + e^{\alpha - (\tau - q/2)^2 - \rho^2}}{1 + e^{\alpha - (\tau + q/2)^2 - \rho^2}} d\rho = \\
&= - \int_0^\infty \rho \ln \frac{1 + e^{\alpha - (\tau + q/2)^2 - \rho^2}}{1 + e^{\alpha - (\tau - q/2)^2 - \rho^2}} d\rho = -L(\tau + q/2, \tau - q/2).
\end{aligned}$$

Hence, integral J_1 equals

$$\begin{aligned}
J_1 &= \int_{-\infty}^\infty \frac{L(\tau + q/2, \tau - q/2)}{q\tau - \Omega} \left[\frac{\tau - q/2}{q\tau - q^2/2 - \Omega} - \frac{\tau + q/2}{q\tau + q^2/2 - \Omega} \right] d\tau = \\
&= q\Omega \int_{-\infty}^\infty \frac{L(\tau + q/2, \tau - q/2) d\tau}{(q\tau - \Omega)[(q\tau - \Omega)^2 - q^4/4]}.
\end{aligned}$$

Let us calculate the second integral from (4.10)

$$\begin{aligned}
J_4 &= \frac{1}{2} \int_{-\infty}^\infty \ln \frac{1 + e^{\alpha - (\tau + q)^2}}{1 + e^{\alpha - (\tau - q)^2}} \frac{\tau d\tau}{q\tau - \Omega} = \\
&= \frac{1}{2q} \int_{-\infty}^\infty \ln \frac{1 + e^{\alpha - (\tau + q)^2}}{1 + e^{\alpha - (\tau - q)^2}} \left(1 + \frac{\Omega}{q\tau - \Omega} \right) d\tau = \\
&= \frac{\Omega}{2q} \int_{-\infty}^\infty \ln \frac{1 + e^{\alpha - (\tau + q)^2}}{1 + e^{\alpha - (\tau - q)^2}} \frac{d\tau}{q\tau - \Omega} = \\
&= \frac{\Omega}{2q} \int_{-\infty}^\infty \ln(1 + e^{\alpha - x^2}) \left[\frac{1}{qx - q^2 - \Omega} - \frac{1}{qx + q^2 - \Omega} \right] dx = \\
&= \Omega q \int_{-\infty}^\infty \frac{\ln(1 + e^{\alpha - x^2}) dx}{(qx - \Omega)^2 - q^4/4} dx.
\end{aligned}$$

Finally, integral (4.10) equals

$$J_{\text{quant}}(\Omega, q) = -\frac{1}{16\pi l_0(\alpha)\Omega^2} \left[\int_{-\infty}^\infty \frac{L(\tau + q/2, \tau - q/2) d\tau}{(q\tau - \Omega)[(q\tau - \Omega)^2 - q^4/4]} + \right.$$

$$+ \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha-x^2})dx}{(qx - \Omega)^2 - q^4/4} dx \Big]. \quad (4.11)$$

At calculation singular integral from (4.10), which not writing out let us designate through $I(\Omega, q)$, it is necessary to take advantage known Landau rule. Then

$$I(\Omega, q) = \text{Re } I(\Omega, q) + i \text{Im } I(\Omega, q).$$

Here

$$\begin{aligned} \text{Re } I(\Omega, q) = & \text{V.p.} \int_{-\infty}^{\infty} \frac{L(\tau + q/2, \tau - q/2)d\tau}{(q\tau - \Omega)[(q\tau - \Omega)^2 - q^4/4]} + \\ & + \text{V.p.} \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha-x^2})dx}{(qx - \Omega)^2 - q^4/4} dx, \end{aligned}$$

Symbol V.p. means principal value of integral also,

$$\begin{aligned} \text{Im } I(\Omega, q) = & -\frac{\pi}{q^4} \left\{ (q^2 - 2\Omega)L\left(\frac{q}{2} + \frac{\Omega}{q}, -\frac{q}{2} + \frac{\Omega}{q}\right) + \right. \\ & + (q^2 + 2\Omega)L\left(-\frac{q}{2} + \frac{\Omega}{q}, \frac{q}{2} + \frac{\Omega}{q}\right) + \\ & \left. + 2\Omega \left[L\left(\frac{\Omega}{q} + q, \frac{\Omega}{q}\right) - L\left(\frac{\Omega}{q} - q, \frac{\Omega}{q}\right) \right] + \frac{\Omega q^2}{2} \ln \frac{1 + e^{\alpha-(\Omega/q+q)^2}}{1 + e^{\alpha-(\Omega/q-q)^2}} \right\}. \end{aligned}$$

Considering antisymmetry of function $L(X, Y)$ on the variables ($L(X, Y) = -L(Y, X)$), we will simplify expression for an imaginary part of density of the dimensionless current. It is as a result received, that

$$\begin{aligned} \text{Im } I(\Omega, q) = & -\frac{\pi\Omega}{q^4} \left[-4L\left(\frac{q}{2} + \frac{\Omega}{q}, -\frac{q}{2} + \frac{\Omega}{q}\right) + \right. \\ & \left. + 2\Omega \left[L\left(\frac{\Omega}{q} + q, \frac{\Omega}{q}\right) - L\left(\frac{\Omega}{q} - q, \frac{\Omega}{q}\right) \right] + \frac{\Omega q^2}{2} \ln \frac{1 + e^{\alpha-(\Omega/q+q)^2}}{1 + e^{\alpha-(\Omega/q-q)^2}} \right]. \end{aligned}$$

Equality (4.10) for density of a longitudinal current we will present into the vector form

$$\mathbf{j}^{\text{quant}} = J_{\text{quant}}(\Omega, q) \sigma_{l, tr} k \mathbf{E}_{tr}^2.$$

Let us show, that at small values of wave number density longitudinal current both in quantum and in classical plasma coincide.

According to (4.7) at small q after linearization $f_0(P_x \pm q)$ it is received

$$j_x^{\text{quant}} = \frac{2e^3 p_T^3 A_y^2 q}{(2\pi\hbar)^3 c^2 m^2 v_T \Omega} \int g(P) P_x^2 d^3 P = \frac{2\pi e^3 p_T^3 A_y^2 l_0(\alpha)}{(2\pi\hbar)^3 c^2 m^2 \omega} k.$$

Let us take advantage of relation of thermal wave number with the numerical density, and also relation of potential and intensity of electromagnetic field. We receive, that

$$j_x^{\text{quant}} = -\frac{1}{8\pi} \cdot \frac{e}{m\omega} \left(\frac{\omega_p}{\omega}\right)^2 k E_y^2. \quad (4.11)$$

Expression (4.11) in accuracy coincides with (1.12). These expressions let us copy in the vector kind

$$\mathbf{j}^{\text{long}} = -\frac{1}{8\pi} \cdot \frac{e}{m\omega} \left(\frac{\omega_p}{\omega}\right)^2 \mathbf{k} E_{tr}^2.$$

5 CONCLUSIONS

Graphic investigations of sizes of density of longitudinal current we will spend for case: chemical potential equals to zero ($\alpha = 0$). Curves 1 answer to classical plasma, curves 2 to quantum.

On figs. 1 and 2 we will present behaviour real (fig. 1) and imaginary (fig. 2) parts of density of dimensionless longitudinal current at $\Omega = 0.5$ Depending on dimensionless wave number q . At decrease parametre Ω curves 1 and 2 approach and become indiscernible.

On fig. 3 and 4 we will represent behaviour real (fig. 3) and imaginary (fig. 4) density parts of longitudinal current depending on dimensionless wave number q in the case $\Omega = 1$. From these drawings it is visible, that with growth dimensionless frequency of oscillations of an electromagnetic field of ordinate graphics quickly decrease.

On fig. 5 and 6 we will represent behaviour real (fig. 5) and imaginary (fig. 6) parts of longitudinal current depending on dimensionless frequency of oscillations of electromagnetic field Ω in the case $q = 0.3$. At reduction

dimensionless wave number q curves 1 and 2 approximate and at small q practically coincide.

On figs. 7–10 we consider behaviour of classical plasmas.

On fig. 7 and 8 the behaviour real (fig. 7) and imaginary (fig. 8) is represented parts of longitudinal current depending on dimensionless wave number, Ω . Curves 1,2,3 accordingly answer values dimensionless chemical potential $\alpha = -5, 0, 3$.

On fig. 9 and 10 we will represent behaviour real (fig. 9) and imaginary (fig. 10) parts of longitudinal current depending on dimensionless wave number q in a case $\Omega = 1$. Curves 1,2,3 answer according to values of the dimensionless chemical potential $\alpha = -5, 0, 3$.

In the present work the account of nonlinear character of interaction electromagnetic field with classical and quantum plasma is considered. It has appeared, that the account of nonlinearity of an electromagnetic field finds out generating of an electric current, orthogonal to a direction of electromagnetic field.

Further authors purpose to consider new problems about oscillations plasmas and about skin-effect with use square-law onto potential of expansion of distribution function.

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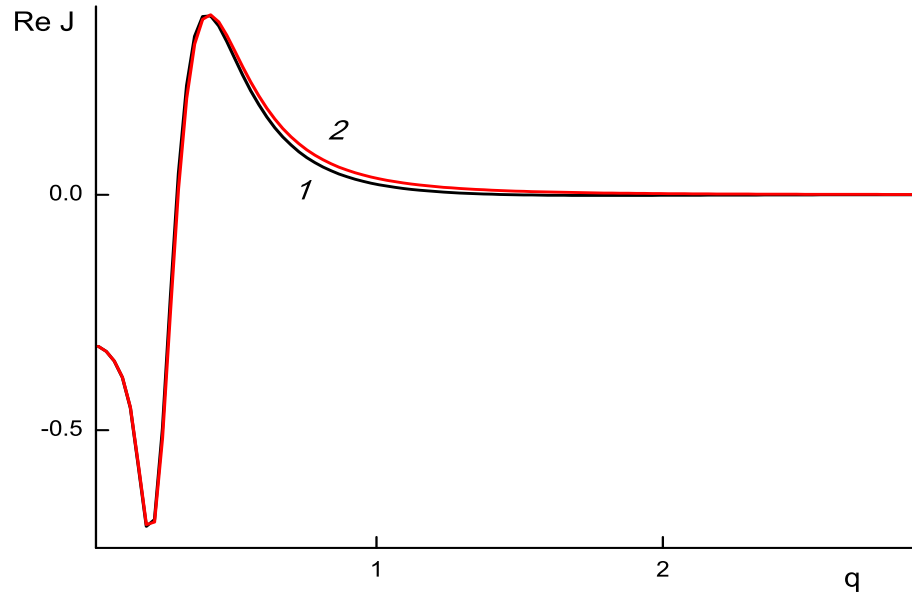


Fig. 1. Real part of density of dimensionless current, $\Omega = 0.5$.

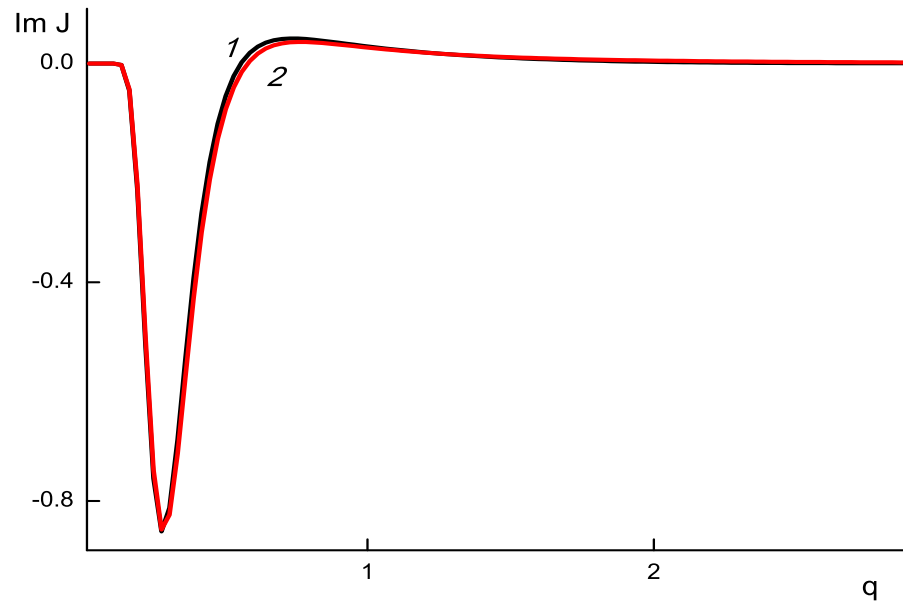


Fig. 2. Imaginary part of density of dimensionless current, $\Omega = 0.5$.

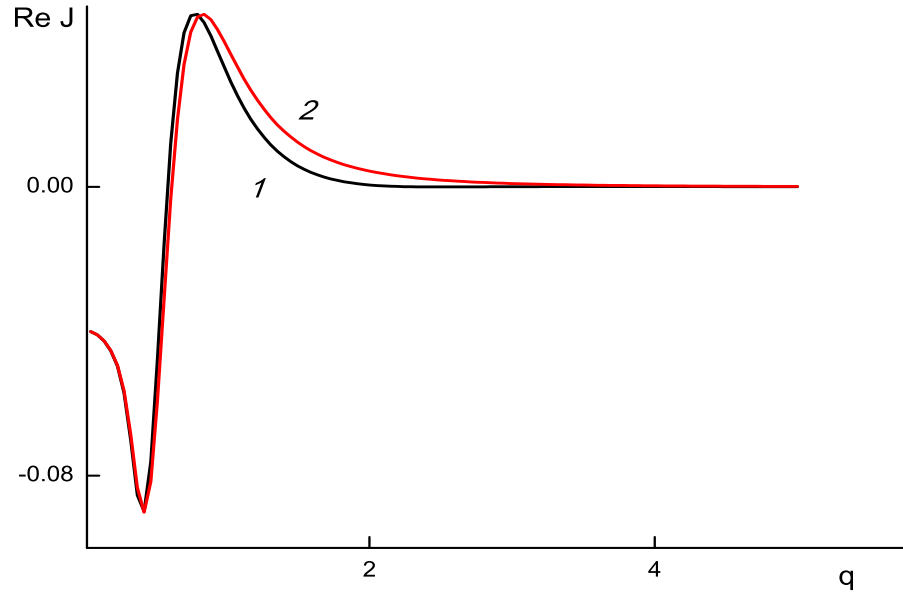


Fig. 3. Real part of density of dimensionless current, $\Omega = 1$.

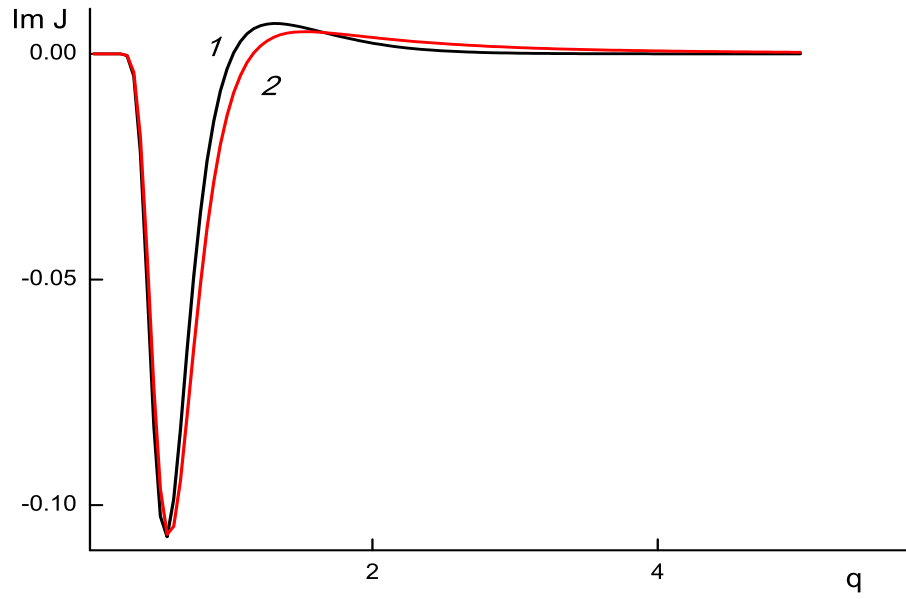


Fig. 4. Imaginary part of density of dimensionless current, $\Omega = 1$.

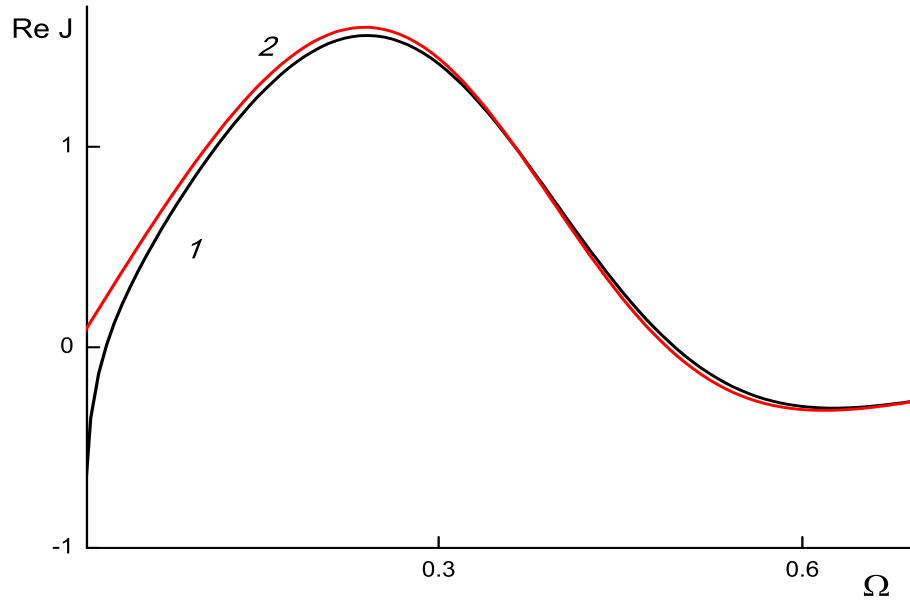


Fig. 5. Real part of density of dimensionless current, $q = 0.3$.

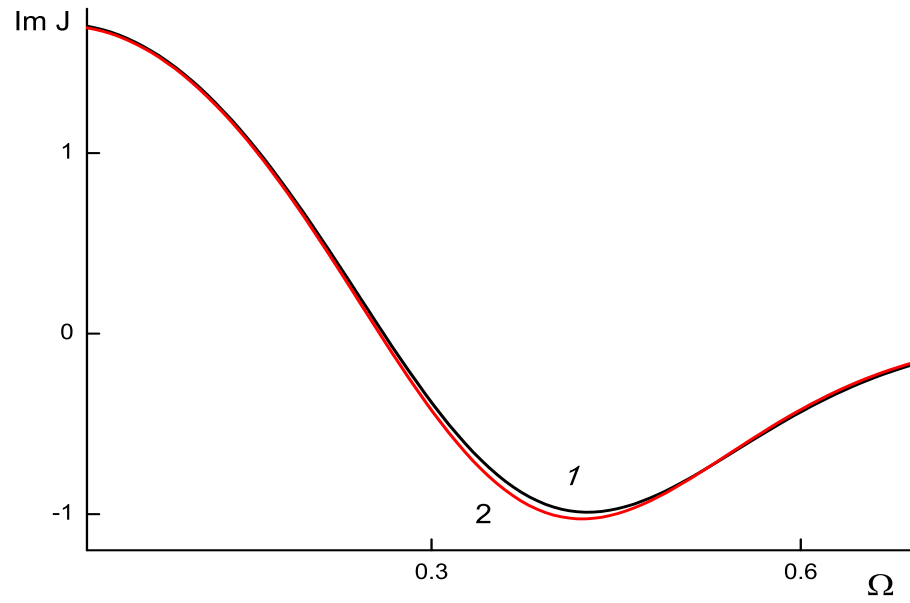


Fig. 6. Imaginary part of density of dimensionless current, $q = 0.3$.

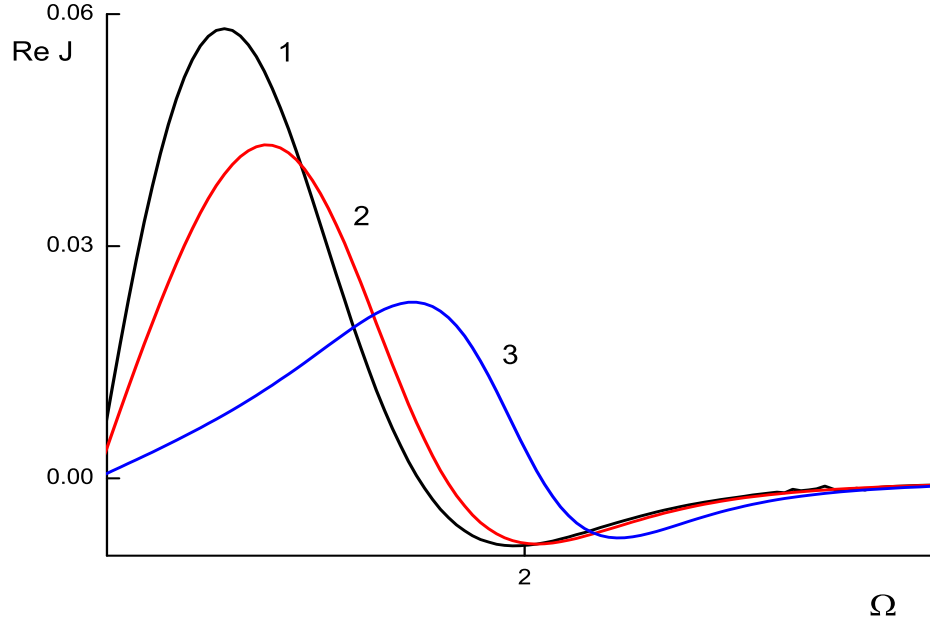


Fig. 7. Real part of density of dimensionless current in classical plasma, $q = 1$. Curves 1,2,3 correspond to values of dimensionless chemical potential $\alpha = -5, 0, 3$.

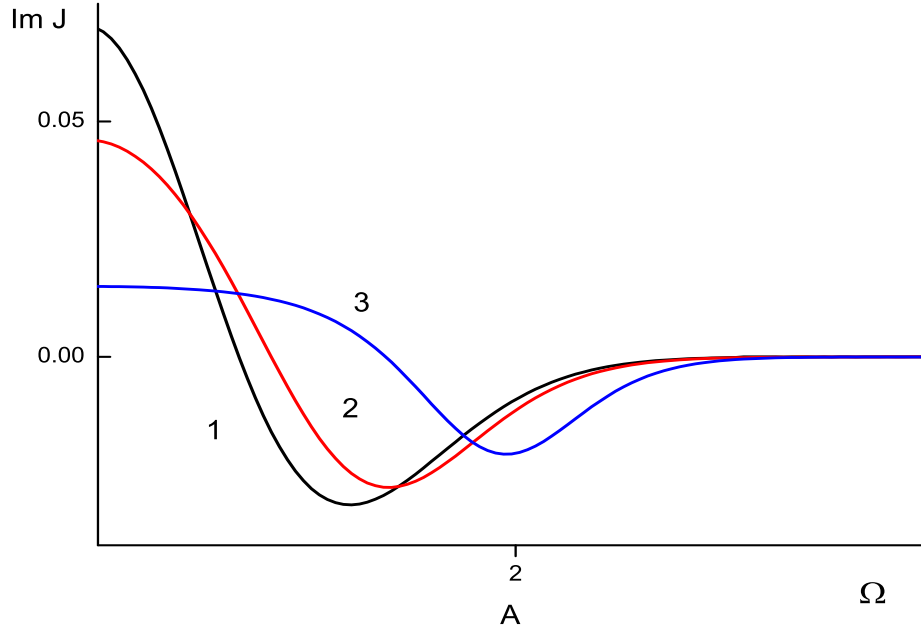


Fig. 8. Imaginary part of density of dimensionless current in classical plasma, $q = 1$. Curves 1,2,3 correspond to values of dimensionless chemical potential $\alpha = -5, 0, 3$.

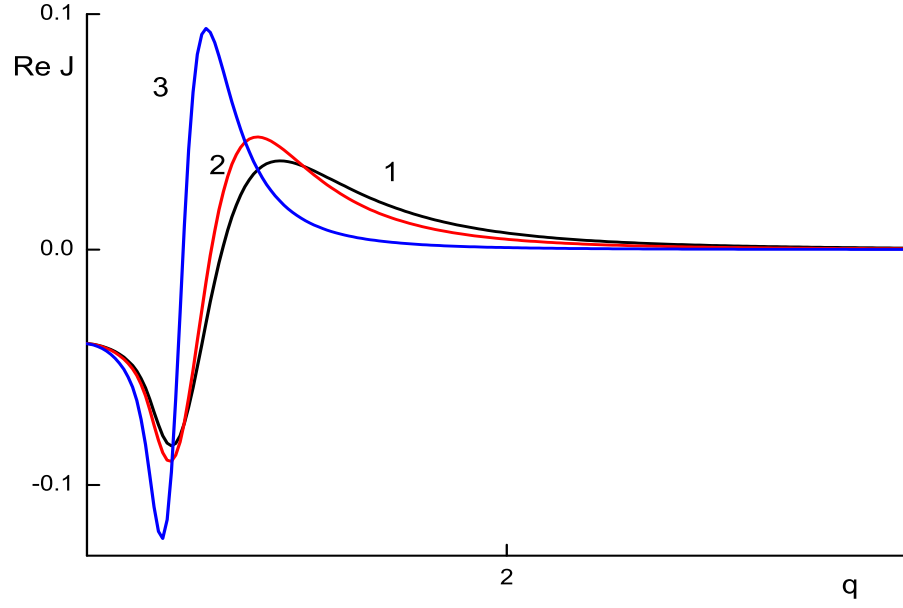


Fig. 9. Real part of density of dimensionless current in classical plasma, $\Omega = 1$. Curves 1,2,3 correspond to values of dimensionless chemical potential $\alpha = -5, 0, 3$.

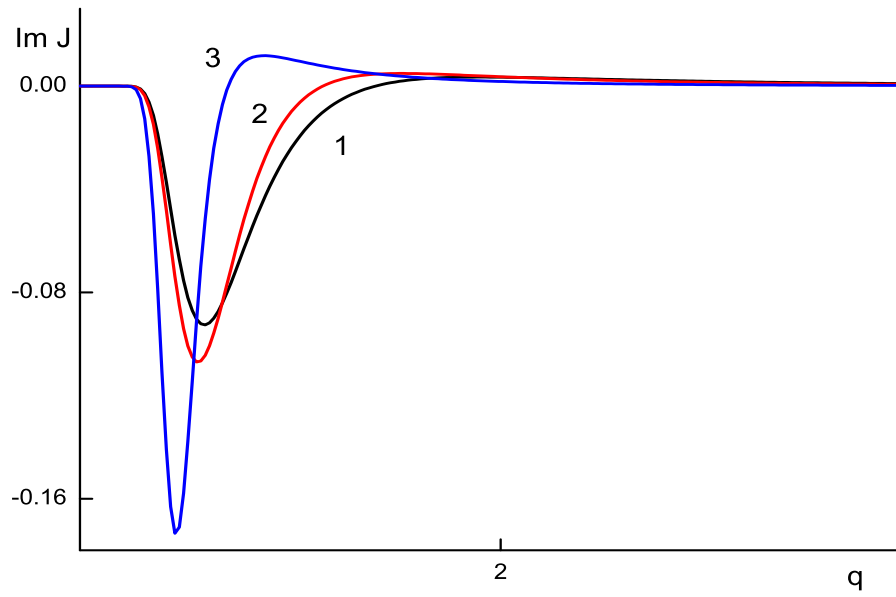


Fig. 10. Imaginary part of density of dimensionless current in classical plasma, $\Omega = 1$. Curves 1,2,3 correspond to values of dimensionless chemical potential $\alpha = -5, 0, 3$.